Modeling the business and financial cycle in a multivariate structural time series model

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Main contributions

- Novel approach to **simultaneously** extract a short-term cycle and a medium-term cycle from a panel of macroeconomic and financial time series
- and **simultaneously** estimate co-cyclicality of cycles
- and **simultaneously** mix time-series with monthly and quarterly frequencies

Motivation

- Several papers document existence of medium-term macroeconomic cycles (e.g. Comin and Gertler, 2006 and Correa-López and de Blas, 2012);
- Since the Global Financial Crisis, the policy debate has increasingly paid attention to the concept of the financial cycle. (e.g. Borio, 2014 and Drehman et al., 2012)
- There has also been a fast growing literature exploring ways to estimate financial cycles and analyzing their properties.

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Conclusion

1. We find strong evidence for the existence of a separate short-term cycle and medium-term cycle in macroeconomic and financial variables in industrialized countries
2. Co-movement between macroeconomic and financial variables limited to the medium-term
3. Strong concordance between the medium-term cycles of house prices and GDP. much less between credit and GDP
4. bulk of the estimated movements driven by domestic rather than global factors (see paper)

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Presentation Outline

1. Estimation method
2. Outcomes
3. Conclusion

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Unobserved Component (UC) models

- We apply the Kalman filter-smoother to an unobserved components time series model to extract multiple cycles, see Harvey (1989) and Durbin and Koopman (2012) for an overview.
- This approach has been applied to business cycle analysis, extracting **one** cycle, see e.g. Valle e Azevedo et al., 2006; Creal et al., 2010.
- Koopman and Lucas (2005) is one of the few papers extracting **two** cycles. They extract cycles from asset prices in the Unites States.

Advantages of UC models and the Kalman filter

- Estimating an unobserved components model allows for **simultaneous extraction** of trend, short-term cycle, medium-term cycle and error term via the Kalman filter/smoother algorithm.
- Since the Kalman filter/smoother is based on a model, researchers have the possibility to use **diagnostics** to estimate the fit and validity of this model and hence the accuracy of their estimates.
- The **cycle frequency is also estimated instead of predetermined** as in non-parametric filters and turning point methods. This feature is especially convenient for estimating financial cycles, since there is no broad consensus yet on their characteristics.

Multivariate Unobserved Component Model Framework

Model specification as in Koopman and Lucas (2005):

$$y_t = \mu_t + A\gamma_t + B\psi_t + \varepsilon_t, \qquad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \Sigma_\varepsilon), \tag{1}$$

y_t: time series in a panel with length N, μ_t: **long-term** trend, γ_t: **short-term** cycle, ψ_t: **medium-term** cycle, ε_t : noise.

Unobserved components μ_t, γ_t and ψ_t are assumed to represent unique dynamic processes and are **independent** of each other.

Covariances between the disturbances are non-zero. The loading matrices A and B reveal whether there is co-cyclicality between the time series in the panel. Trend is modeled as an integrated random walk process:

$$\begin{aligned} \mu_{t+1} &= \mu_t + \beta_t, \\ \beta_{t+1} &= \beta_t + \zeta_t, \qquad \zeta_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \Sigma_\zeta), \end{aligned} \tag{2}$$

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Modeling the cycles

The short-term cycle (γ_t) and the medium-term cycle (ψ_t) are specified as a restricted trigonometric processes as proposed by Harvey(1989). Consider ψ_t, i.e.:

$$\begin{pmatrix} \psi_{t+1} \\ \psi^*_{t+1} \end{pmatrix} = \phi_\psi \begin{bmatrix} \cos\lambda_\psi & \sin\lambda_\psi \\ -\sin\lambda_\psi & \cos\lambda_\psi \end{bmatrix} \begin{pmatrix} \psi_t \\ \psi^*_t \end{pmatrix} + \begin{pmatrix} \omega_t \\ \omega^*_t \end{pmatrix},$$

$$\begin{pmatrix} \omega_t \\ \omega^*_t \end{pmatrix} \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2_{i,\omega}), \tag{3}$$

where, ψ_t = (medium-term) cycle, ψ^*_t = 'first derivative' of the (medium-term) cycle, λ_ψ = cycle frequency, ϕ_ψ = persistence parameter ($0 < \phi_\psi < 1$), ω_t = disturbance term. The length of ψ_t is given by $p = 2\pi/\lambda_\psi$.

For identification of the cycle disturbance variances, A and B in Eq.(1) are restricted to be lower triangular matrices with unity as diagonal elements.

▸ Details

Empirical set-up

We apply the multivariate UC model to extract trends and cycles from the following variables (all in real terms):

- Gross domestic product (GDP)
- House prices (HP)
- Bank credit to private sector (CRED)
- Industrial production index (IP)

Countries analyzed:

- We consider the G7-countries (US, UK, JA, CA, DE, FR, IT) and NL

Period of analysis

- 1970Q1–2015Q1 for GDP, HP & CRED;
- 1970M1–2015M12 for IP.

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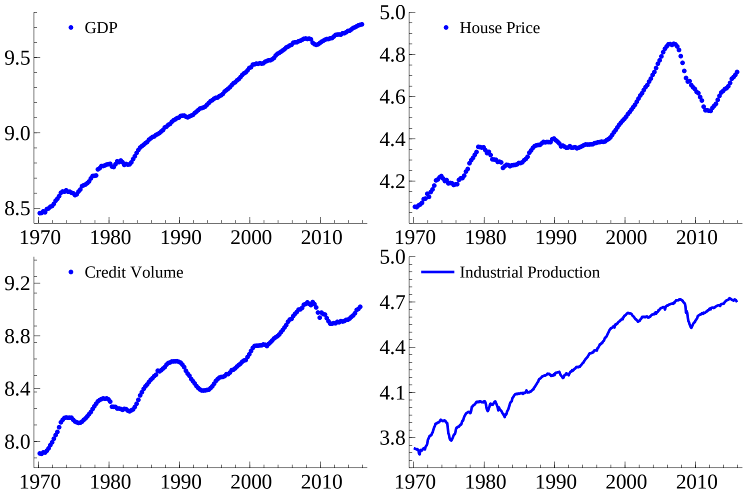
Why two cycles in one model framework?



All US series are deflated, seasonally adjusted and in natural logs.

Results UK, JA, CA, DE, FR, IT, NL

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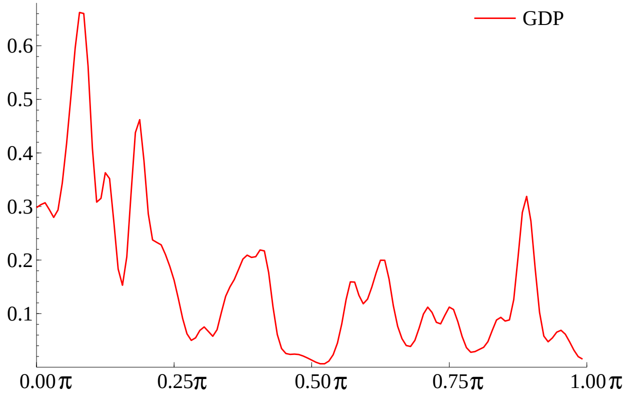
Why two cycles in one model framework?



Estimated spectral densities (US data), series are in log-differences,0.25π translates into a cycle with period of $\frac{2\pi}{0.25\pi} = 8$ quarters (2 years). 0.50π translates into a cycle with period of $\frac{2\pi}{0.25\pi} = 4$ quarters (1 year).

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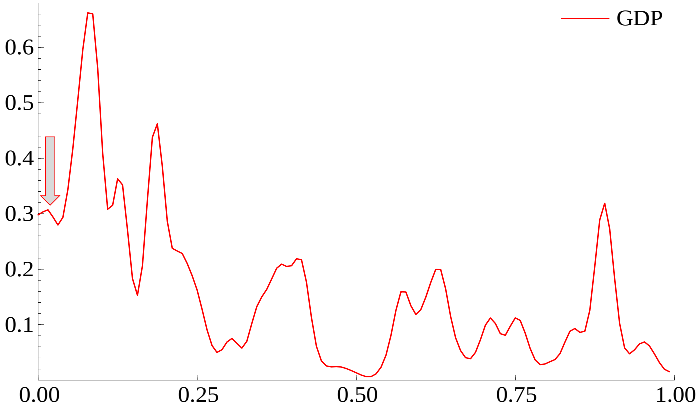
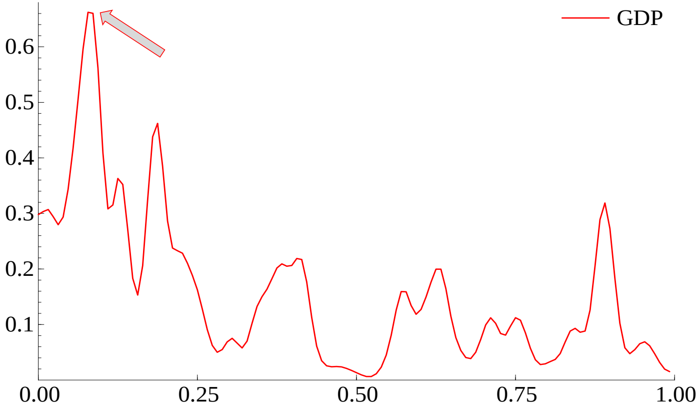
Why two cycles in one model framework?



Estimated spectral densities (US data), series are in log-differences.

The **first** peak in the spectral density of GDP is estimated at approximately 0.02π, which translates into a cycle with period of $\frac{2\pi}{0.02\pi} = 100$ quarters, or 25 years.

Why two cycles in one model framework?



Estimated spectral densities (US data), series are in log-differences.

The **second** peak is at 0.08π, which translates to a period of 25 quarters, or $6\frac{1}{4}$ year. Seems to be related to the business cycle frequency.

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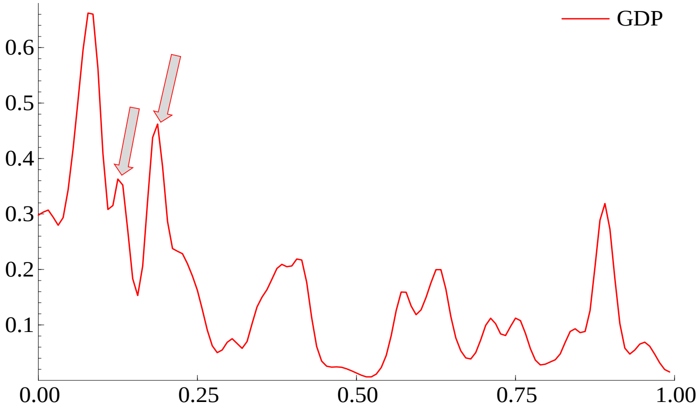
Why two cycles in one model framework?



Estimated spectral densities (US data), series are in log-differences.

The **third** and **fourth** peak occur at 0.13π (15 quarters; 3.8 years) and 0.19π (10 quarters; 2.6 years). Most business cycle frequencies have period of $6\frac{1}{4}$ years.

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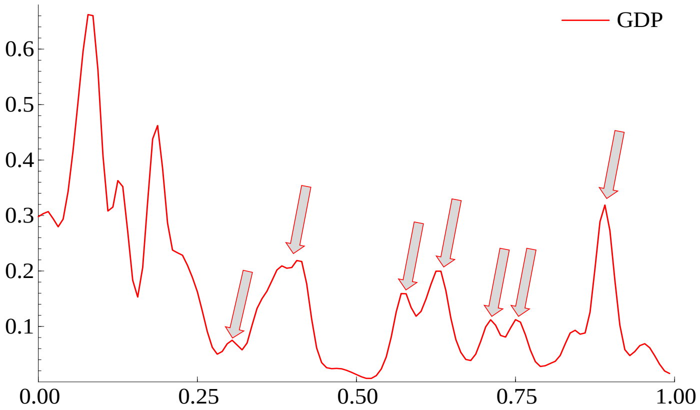
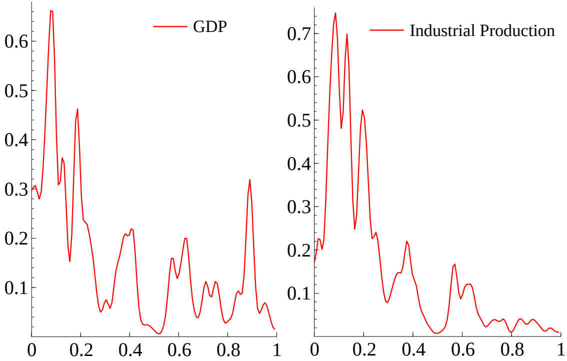
Why two cycles in one model framework?



Estimated spectral densities (US data), series are in log-differences.

There are some local peaks in the spectral density above approximately 0.25π (or 2 years). For our study these fluctuations are not so much of interest (noise).

Why two cycles in one model framework?



Estimated spectral densities (US data), series are in log-differences.

Spec. dens. IP very similar to the spec. dens. of GDP. Small peak at cycle-length of $\pm$ 25 years, and peaks at cycles of approximately 6, 4 and 3 years.

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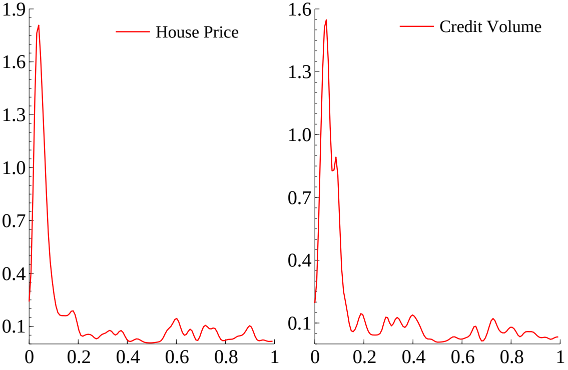
Why two cycles in one model framework?



Estimated spectral densities (US data), series are in log-differences.

Spec. dens. HP and CRED **quite different** from GDP and IP. Former show large peaks at cycle-length of approximately 13 years. Not much cycl. movement at higher freq's.

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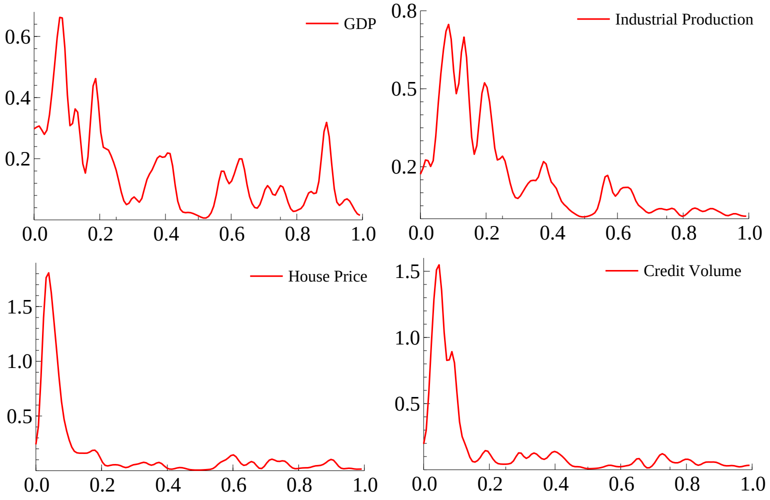
Why two cycles in one model framework?



Estimated spectral densities (US data), series are in log-differences.

Results UK, JA, CA, DE, FR, IT, NL

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Why two cycles in one model framework?

- Besides spectral densities we perform statistical likelihood ratio (LR) test, following Rünstler and Vlekke (2018) and Galati et al. (2016).
- We conclude that in almost all countries and variables our four variables have two cycles (exceptions: GDP in NL and IP in UK)
- Our main conjecture from analyzing the spectral densities and formal testing:
  – **medium-term** frequencies are dominant in the house price & credit volume variables
  – **short-term** fluctuations are dominant for GDP & industrial production

Model applied to our dataset

Our model once more:

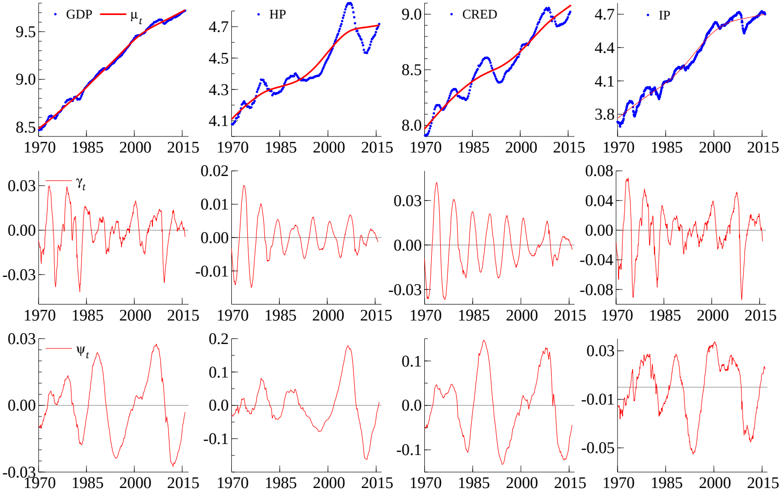
$$y_t = \begin{bmatrix} y_t^{\text{GDP}} \\ y_t^{\text{HP}} \\ y_t^{\text{CRED}} \\ y_t^{\text{IP}} \end{bmatrix} = \begin{bmatrix} \text{real GDP (GDP)} \\ \text{real House Price (HP)} \\ \text{real Credit Volume (CRED)} \\ \text{real Industrial Production (IP)} \end{bmatrix},$$

$$= \mu_t + A\gamma_t + B\psi_t + \varepsilon_t, \quad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \Sigma_\varepsilon),$$

for $t = 1, \ldots, T$, with μ_t is the trend, γ_t is the short-term cycle and ψ_t is the medium-term cycle component. The model is cast in state space form and estimated using the maximum likelihood method.

Details state space model

Graphical representation outcomes for the United States



Estimated trend (μ_t), short-term cycle (γ_t) and medium-term cycle (ψ_t) for (US data) GDP, HP, CRED and IP. Series are in logs.

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Results UK, JA, CA, DE, FR, IT, NL

Properties and commonality of the short-term cycle λ_γ

Parameter estimates of multivariate UCTSM for short-term cycle

	US	UK	JA	CA	DE	FR	IT	NL
Properties cycles								
p_γ	5.4	6.7	3.3	7.1	4.5	3.6	3.8	7.9
ϕ_γ	0.99	0.99	0.98	0.99	0.97	0.98	0.97	0.99
Loading matrix A								
vs. GDP // 1st column loading matrices								
HP	-0.02	1.79	0.48	-0.14	-0.09	0.52	-0.38	-0.06
CRED	0.25	0.20	0.01	0.94*	0.07	0.08	0.33	0.88*
IP	1.82***	1.11**	3.26***	1.97***	2.82***	3.14***	2.54***	1.45**
vs. HP // 2nd column loading matrices								
CRED	1.26	0.30	-0.04	0.25	-0.57	-0.13	0.00	-0.16
IP	1.51	0.11	0.43	0.02	0.34	0.86	-0.10	0.03
vs. CRED // 3nd column loading matrices								
IP	-0.07	0.27	-1.99	-0.19	-0.46	2.31	1.81	0.53

The table reports the estimates of persistence ϕ_γ and the period p_γ in years ($p = 2\pi/\lambda$), respectively Significant estimates in these matrices are highlighted in grey. *,** and *** denote statistical significance at the 10%, 5% and 1% level, respectively.

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Properties and commonality of the medium-term cycle λ_ψ

Parameter estimates of multivariate UCTSM for medium-term cycle

	US	UK	JA	CA	DE	FR	IT	NL
Properties cycles								
p_ψ	13.6	18.4	9.2	22.3	9.3	16.2	14.7	23.7
ϕ_ψ	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Loading matrix B								
vs. GDP // 1st column loading matrices								
HP	2.77***	1.19	1.45	8.43	1.80	1.54	19.58	3.56
CRED	4.48	0.80	1.73	-1.98	1.10	1.58	2.78	0.45
IP	1.91	2.42	2.70	3.17	1.69	2.66	2.32	1.71
vs. HP // 2nd column loading matrices								
CRED	-0.25	0.16	0.42	1.28	6.56	-0.07	2.90	0.69
IP	-0.19	-0.36	-0.26	-0.80	-12.64	-0.17	-0.83	-0.48
vs. CRED // 3nd column loading matrices								
IP	-0.58	-0.03	0.06	0.24	0.29	-0.41	2.32	2.62

The table reports the estimates of persistence ϕ_ψ and the period p_ψ in years ($p = 2\pi/\lambda$), respectively Significant estimates in these matrices are highlighted in grey. *,** and *** denote statistical significance at the 10%, 5% and 1% level, respectively.

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Conclusion

1. We find strong evidence for the existence of a separate short-term cycle and medium-term cycle in macroeconomic and financial variables in industrialized countries
2. We present an elegant model to **simultaneously** extract these cycles and estimate their co-cyclicality
3. Co-movement between macroeconomic and financial variables limited to the medium-term
4. Strong concordance between the medium-term cycles of house prices and GDP. much less between credit and GDP

Thank you for your attention!

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Extra Slides

Details of modeling the cycles (1/2)

Remember from Calculus:

- A deterministic cycle with amplitude a and frequency λ_ψ can be written as:

$$\psi_t = a\cos(\lambda_\psi t - b) \qquad a, b, \lambda_\psi, t \in \mathbb{R} \qquad a \neq 0, \lambda_\psi \neq 0$$

- The first partial derivative of ψ_t with respect to $\lambda_\psi t$ equals:

$$\psi_t^* = -a\sin(\lambda_\psi t - b)$$

- The trigonometric identities:

$$\cos(x \pm y) = \cos x \cos y \pm \sin x \sin y$$
$$\sin(x \pm y) = \cos x \sin y \pm \sin x \cos y$$

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Details of modeling the cycles (2/2)

Using the formula's for the first partial derivative and the first trigonometric identity with $x = \lambda_\psi t - b$ and $y = \lambda_\psi$ it follows that,

$$
\begin{aligned}
\psi_{t+1} &= a\cos(\lambda_\psi(t+1) - b) \\
\psi_{t+1} &= a\cos(\lambda_\psi t - b + \lambda_\psi) \\
\psi_{t+1} &= a\cos(\lambda_\psi t - b)\cos\lambda_\psi - a\sin(\lambda_\psi t - b)\sin\lambda_\psi \\
\psi_{t+1} &= \psi_t \cos\lambda_\psi + \psi_t^* \sin\lambda_\psi \\
\psi_{t+1} &= \cos\lambda_\psi \psi_t + \sin\lambda_\psi \psi_t^*
\end{aligned}
$$

Or,

$$
\begin{pmatrix} \psi_{t+1} \\ \psi_{t+1}^* \end{pmatrix} = \phi_\psi \begin{bmatrix} \cos\lambda_\psi & \sin\lambda_\psi \\ -\sin\lambda_\psi & \cos\lambda_\psi \end{bmatrix} \begin{pmatrix} \psi_t \\ \psi_t^* \end{pmatrix} + \begin{pmatrix} \omega_t \\ \omega_t^* \end{pmatrix},
$$

Similarly, using the formula's for the first partial derivative and the second trigonometric identity it can be shown that ψ_{t+1}^* can be written as: $-\sin\lambda_\psi \psi_t + \cos\lambda_\psi \psi_t^*$

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State Space Form

The multivariate components model equations (1) – (4) can be cast into the state space form. The measurement and transition equations are defined as:

$$
\begin{aligned}
y_t &= Z\alpha_t + \varepsilon_t, & \varepsilon_t &\sim N(0, H), \\
\alpha_{t+1} &= T\alpha_t + \nu_t, & \nu_t &\sim N(0, Q),
\end{aligned}
$$

where $y_t = (y_{1,t}, \ldots, y_{N,t})'$, $\varepsilon_t = (\varepsilon_{1,t}, \ldots, \varepsilon_{N,t})'$, and $H = \text{diag}(\sigma^2_{\varepsilon_1}, \ldots, \sigma^2_{\varepsilon_N})$. The state vector α_t is given by the $(6N \times 1)$ vector

$$
\alpha_t = (\mu_t, \beta_t, \gamma_t, \gamma_t^*, \psi_t, \psi_t^*)',
$$

where $\mu_t = (\mu_{1,t}, \ldots, \mu_{N,t})'$ is the long-term trend, $\beta_t = (\beta_{1,t}, \ldots, \beta_{N,t})'$ is the slope, $(\gamma_t, \gamma_t^*)' = (\gamma_{1,t}, \ldots, \gamma_{N,t}, \gamma_{1,t}^*, \ldots, \gamma_{N,t}^*)'$ is the short-term cycle, and $(\psi_t, \psi_t^*)' = (\psi_{1,t}, \ldots, \psi_{N,t}, \psi_{1,t}^*, \ldots, \psi_{N,t}^*)'$ is the medium-term cycle.

Go Back Next

State Space Form

The measurement-transition Z matrix is given by

$$Z = \begin{bmatrix} I_N & 0_{N\times N} & A & 0_{N\times N} & B & 0_{N\times N} \end{bmatrix},$$

with A and B are $(N \times N)$ lower triangular matrices with ones on the diagonal. The state-transition matrix T is given by

$$T = \begin{bmatrix} I_N & I_N & 0_{N\times 2N} & 0_{N\times 2N} \\ 0_{N\times N} & I_N & 0_{N\times 2N} & 0_{N\times 2N} \\ 0_{2N\times N} & 0_{2N\times N} & S & 0_{2N\times 2N} \\ 0_{2N\times N} & 0_{2N\times N} & 0_{2N\times 2N} & L \end{bmatrix},$$

with S and L are $(2N \times 2N)$ matrices defined as:

$$S = \phi_\gamma \begin{bmatrix} \cos\lambda_\gamma I_N & \sin\lambda_\gamma I_N \\ -\sin\lambda_\gamma I_N & \cos\lambda_\gamma I_N \end{bmatrix}, \qquad L = \phi_\psi \begin{bmatrix} \cos\lambda_\psi I_N & \sin\lambda_\psi I_N \\ -\sin\lambda_\psi I_N & \cos\lambda_\psi I_N \end{bmatrix}.$$

State Space Form

The state-disturbance vector ν_t is given by

$$\nu_t = (0_{N\times 1}, \zeta_t, \kappa_t, \kappa_t^*, \omega_t, \omega_t^*)',$$

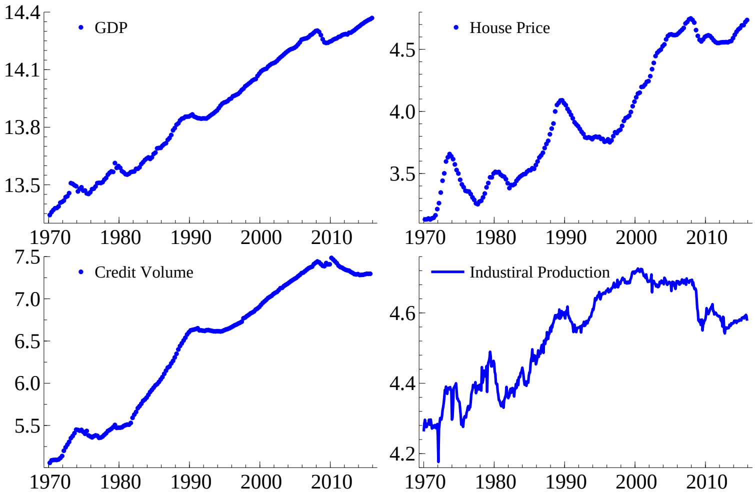
where $\zeta_t = (\zeta_{1,t}, \ldots, \zeta_{N,t})'$ are the slope-disturbances, $(\kappa_t, \kappa_t^*)' = (\kappa_{1,t}, \ldots, \kappa_{N,t}, \kappa_{1,t}^*, \ldots, \kappa_{N,t}^*,)'$ are the short-term cycle disturbances, and $(\omega_t, \omega_t^*)' = (\omega_{1,t}, \ldots, \omega_{N,t}, \omega_{1,t}^*, \ldots, \omega_{N,t}^*,)'$ are the medium-term cycle disturbances.

Lastly, the $(6N \times 6N)$ disturbance matrix Q in the transition equation is defined as:

$$Q = \text{diag}\begin{bmatrix} 0_{N\times N} & \Sigma_\zeta & I_2 \otimes \Sigma_\kappa & I_2 \otimes \Sigma_\omega \end{bmatrix},$$

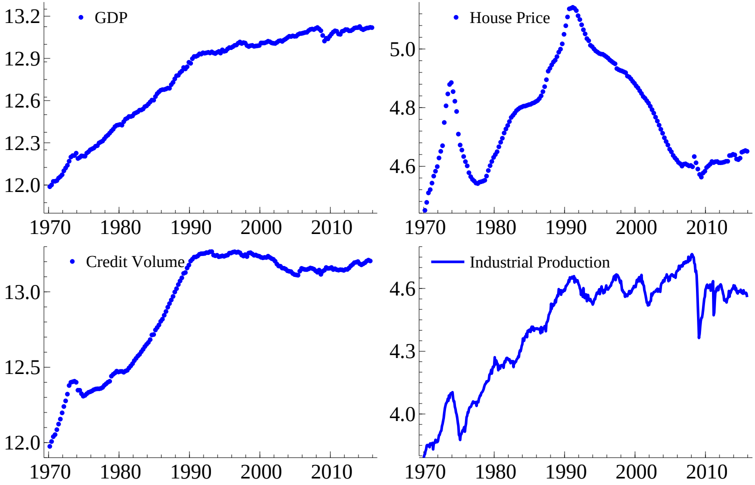
where Σ_ζ is the variance matrix of the slope-disturbances, Σ_κ is the variance matrix of the short-term cycle disturbances, and Σ_ω is the variance matrix of the medium-term cycle disturbances. Σ_ζ is restricted to be diagonal, i.e. $\Sigma_\zeta = \text{diag}(\sigma^2_{\zeta_1}, \ldots, \sigma^2_{\zeta_N})'$ and the signal-to-noise ratio $(\sigma^2_{\zeta_i}/\sigma^2_{\varepsilon_i})$ is fixed for each $i = 1, \ldots, N$.

Why two cycles in one model framework



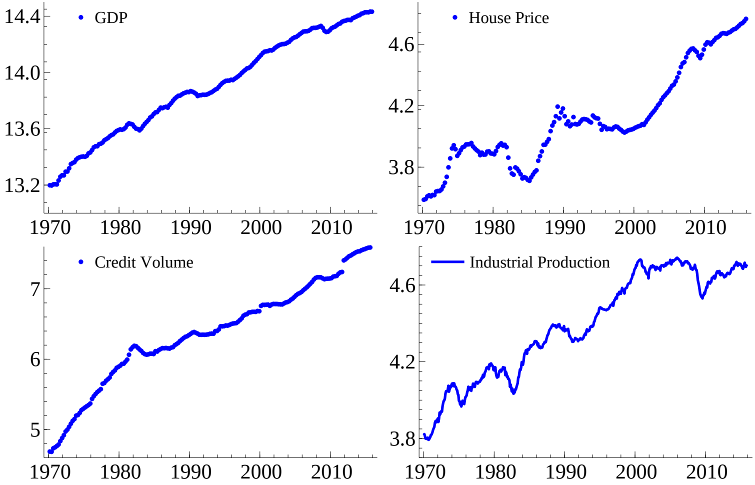
All UK series are deflated, seasonally adjusted and in natural logs.

Why two cycles in one model framework



All JA series are deflated, seasonally adjusted and in natural logs.

Why two cycles in one model framework



All CA series are deflated, seasonally adjusted and in natural logs.

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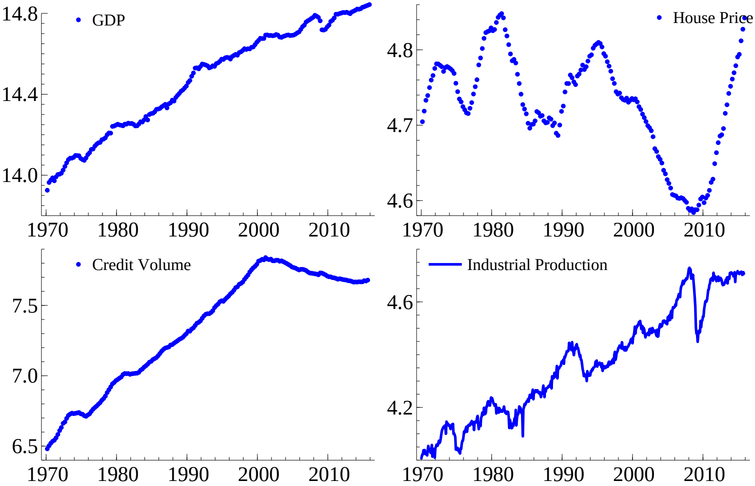
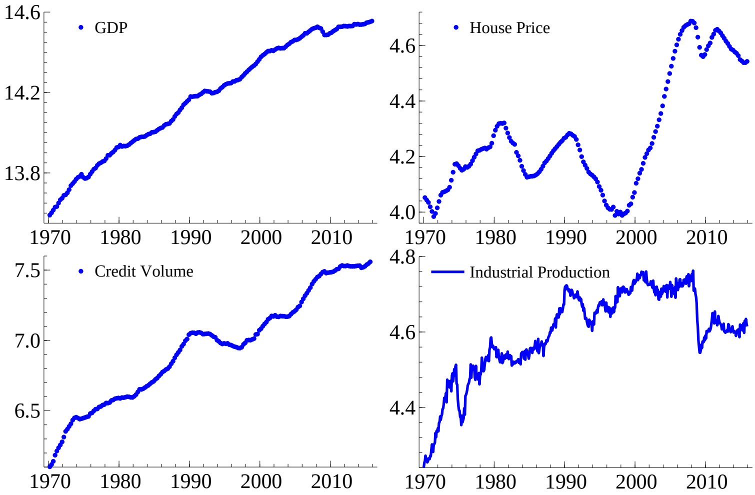
Why two cycles in one model framework



All DE series are deflated, seasonally adjusted and in natural logs.

Go Back
Results France

Why two cycles in one model framework



All FR series are deflated, seasonally adjusted and in natural logs.

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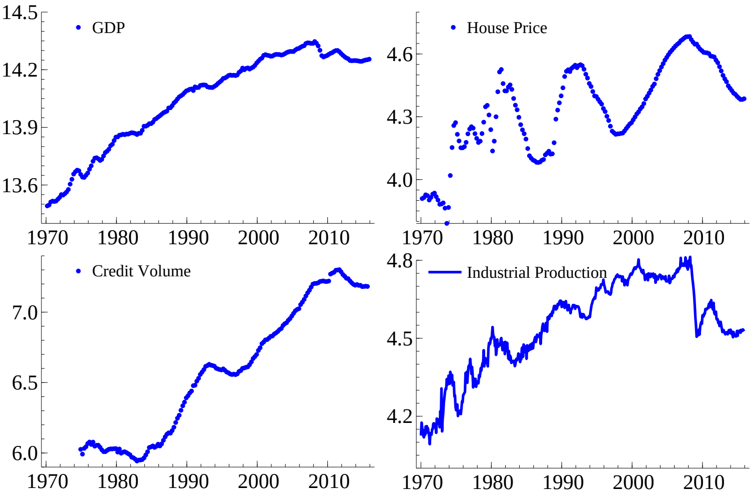
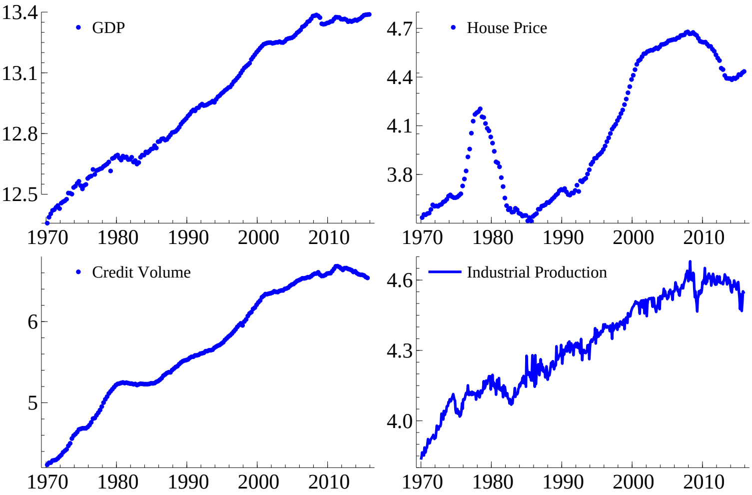
Go Back Results Italy

Why two cycles in one model framework



All IT series are deflated, seasonally adjusted and in natural logs.

Why two cycles in one model framework



All NL series are deflated, seasonally adjusted and in natural logs.

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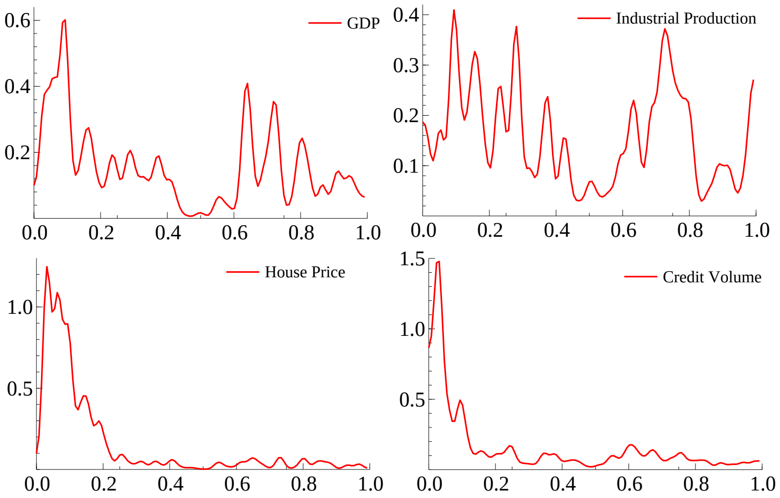
Why two cycles in one model framework



Estimated spectral densities (UK data), series are in log-differences.

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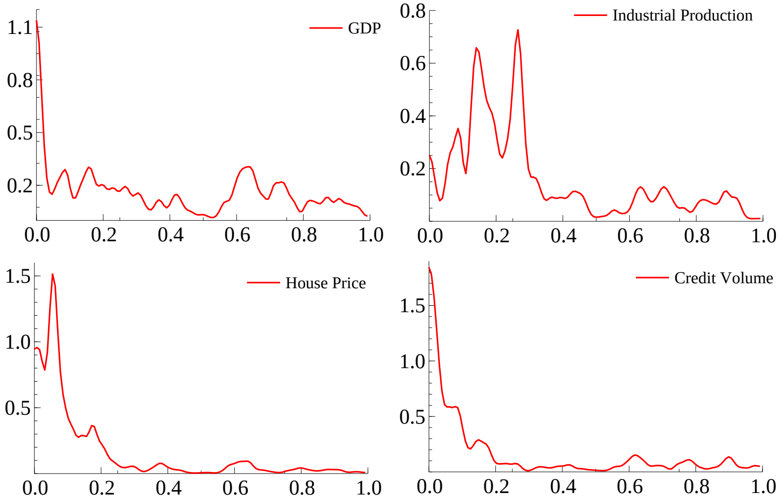
Why two cycles in one model framework



Estimated spectral densities (JA data), series are in log-differences.

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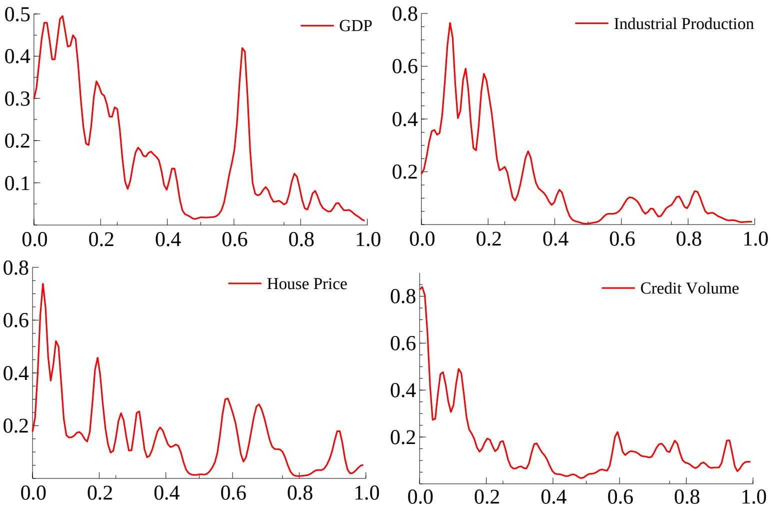
Go Back
Results Canada

Why two cycles in one model framework



Estimated spectral densities (CA data), series are in log-differences.

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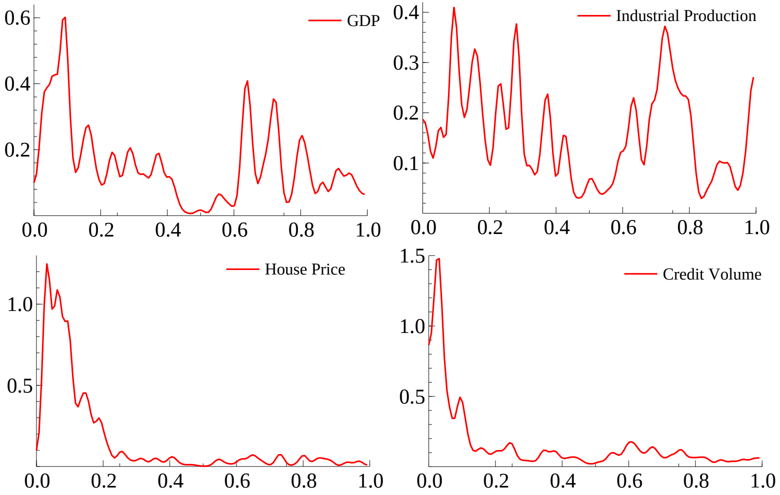
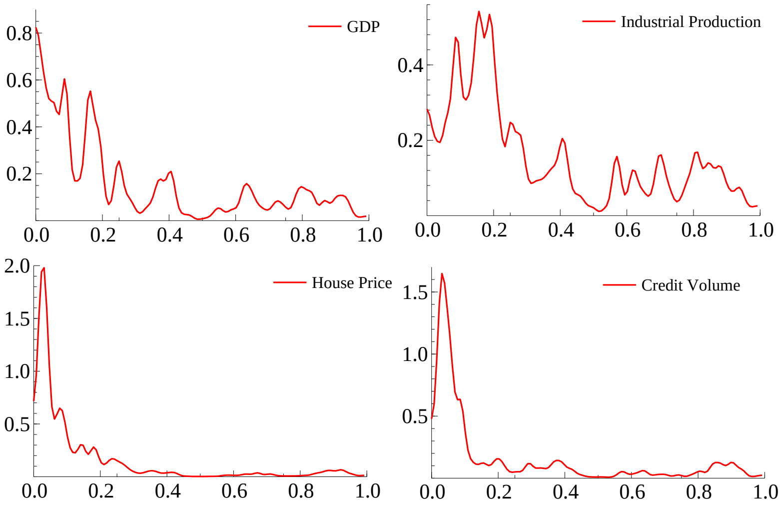
◂ Go Back ▸ Results Germany

Why two cycles in one model framework



Estimated spectral densities (DE data), series are in log-differences.

Why two cycles in one model framework



Estimated spectral densities (FR data), series are in log-differences.

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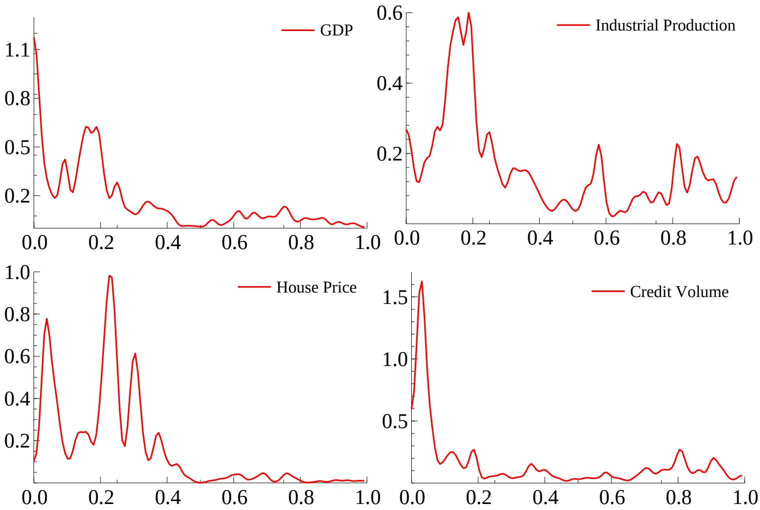
Why two cycles in one model framework



Estimated spectral densities (IT data), series are in log-differences.

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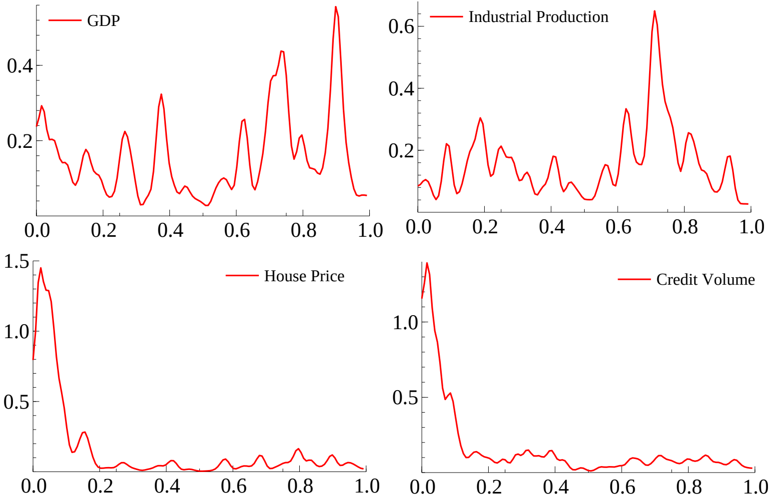
Go Back
Results Netherlands

Why two cycles in one model framework



Estimated spectral densities (NL data), series are in log-differences.

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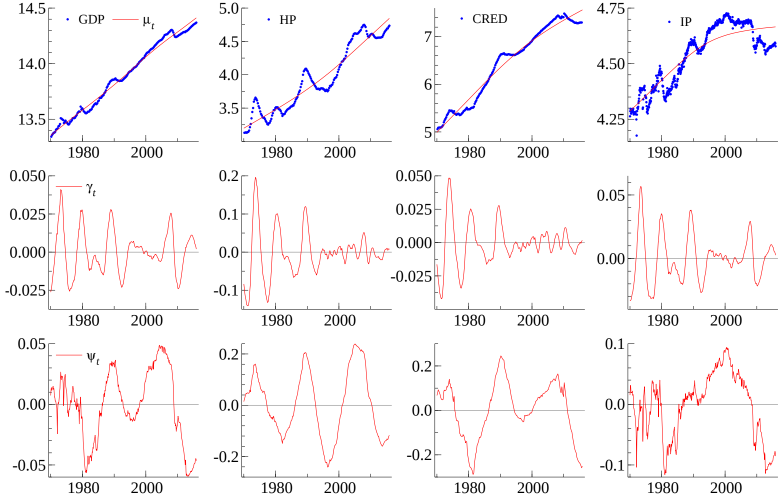
Go Back

Graphical representation outcomes for the United Kingdom



Estimated trend (μ_t), short-term cycle (γ_t) and medium-term cycle (ψ_t) for (UK data) GDP, HP, CRED and IP. Series are in logs.

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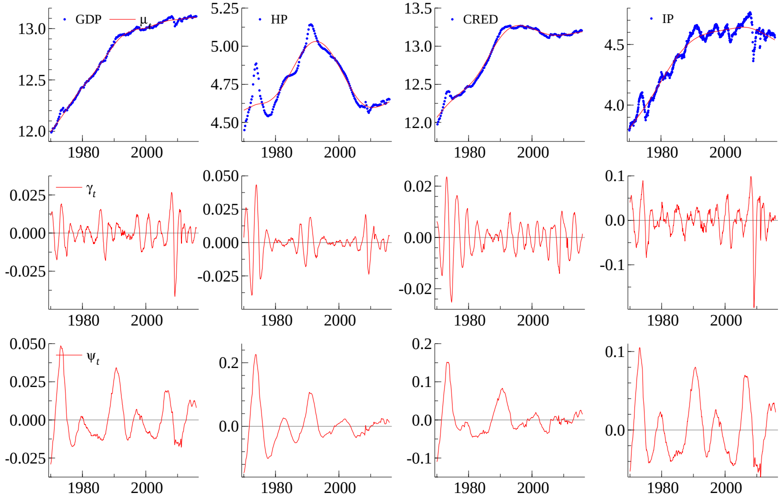
Go Back Results Japan

Graphical representation outcomes for Japan



Estimated trend (μ_t), short-term cycle (γ_t) and medium-term cycle (ψ_t) for (JA data) GDP, HP, CRED and IP. Series are in logs.

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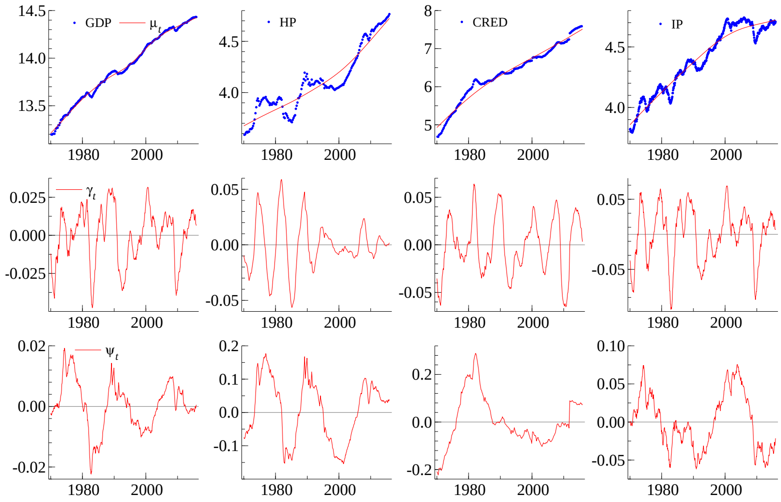
Go Back
Results Canada

Graphical representation outcomes for Canada



Estimated trend (μ_t), short-term cycle (γ_t) and medium-term cycle (ψ_t) for (CA data) GDP, HP, CRED and IP. Series are in logs.

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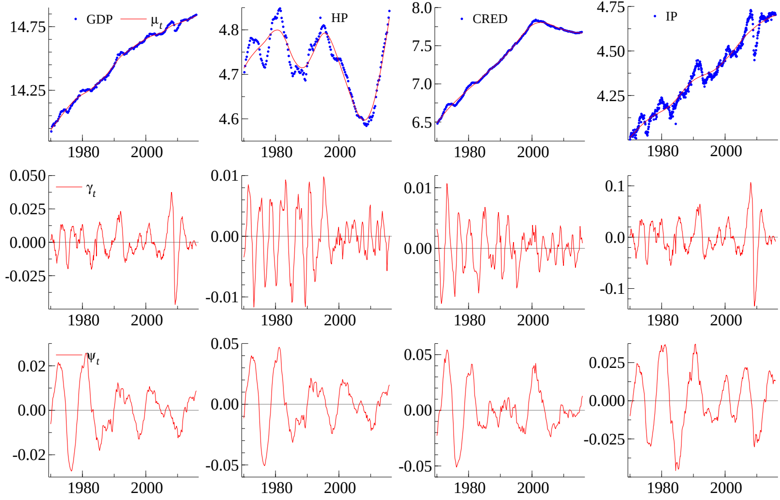
Go Back
Results Germany

Graphical representation outcomes for Germany



Estimated trend (μ_t), short-term cycle (γ_t) and medium-term cycle (ψ_t) for (DE data) GDP, HP, CRED and IP. Series are in logs.

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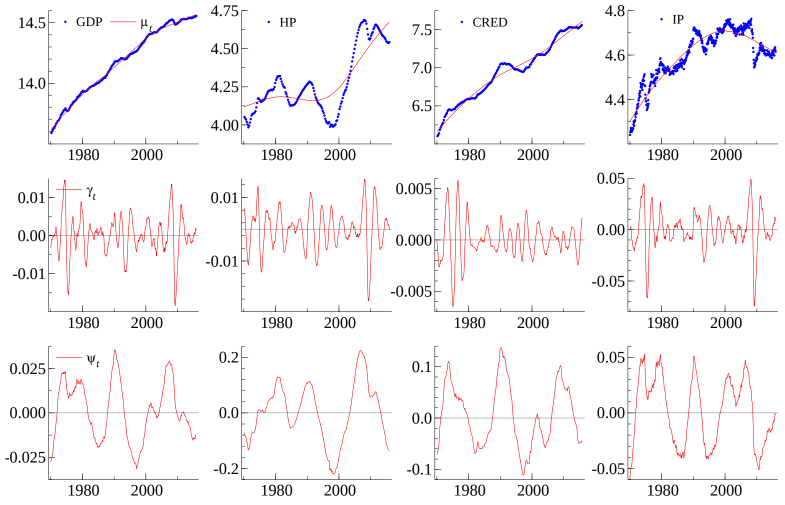
Go Back
Results France

Graphical representation outcomes for France



Estimated trend (μ_t), short-term cycle (γ_t) and medium-term cycle (ψ_t) for (FR data) GDP, HP, CRED and IP. Series are in logs.

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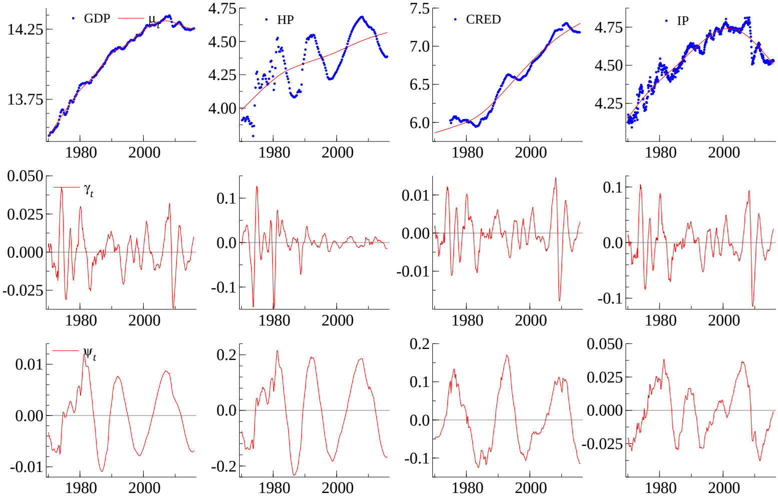
Go Back
Results Italy

Graphical representation outcomes for Italy



Estimated trend (μ_t), short-term cycle (γ_t) and medium-term cycle (ψ_t) for (IT data) GDP, HP, CRED and IP. Series are in logs.

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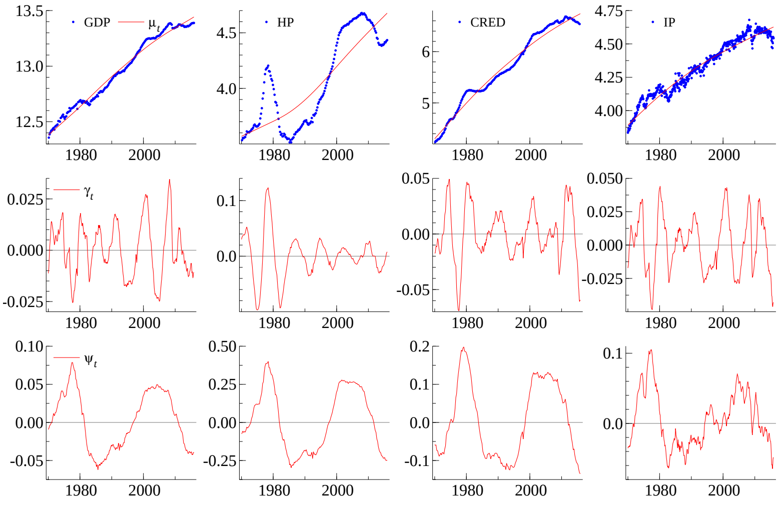
Go Back
Results Netherlands

Graphical representation outcomes for the Netherlands



Estimated trend (μ_t), short-term cycle (γ_t) and medium-term cycle (ψ_t) for (NL data) GDP, HP, CRED and IP. Series are in logs.

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Go Back